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DIRTY PAPER CODING AND DISTRIBUTED SOURCE CODING

TWO VIEWS OF COMBINED SOURCE AND CHANNEL CODING

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FEEC - Unicamp

Summary

- INTRODUCTION

- DIRTY PAPER CODING
  - CODING FOR MEMORIES WITH DEFECTS
  - PARTITIONED LINEAR BLOCK CODES
  - COSET CODES

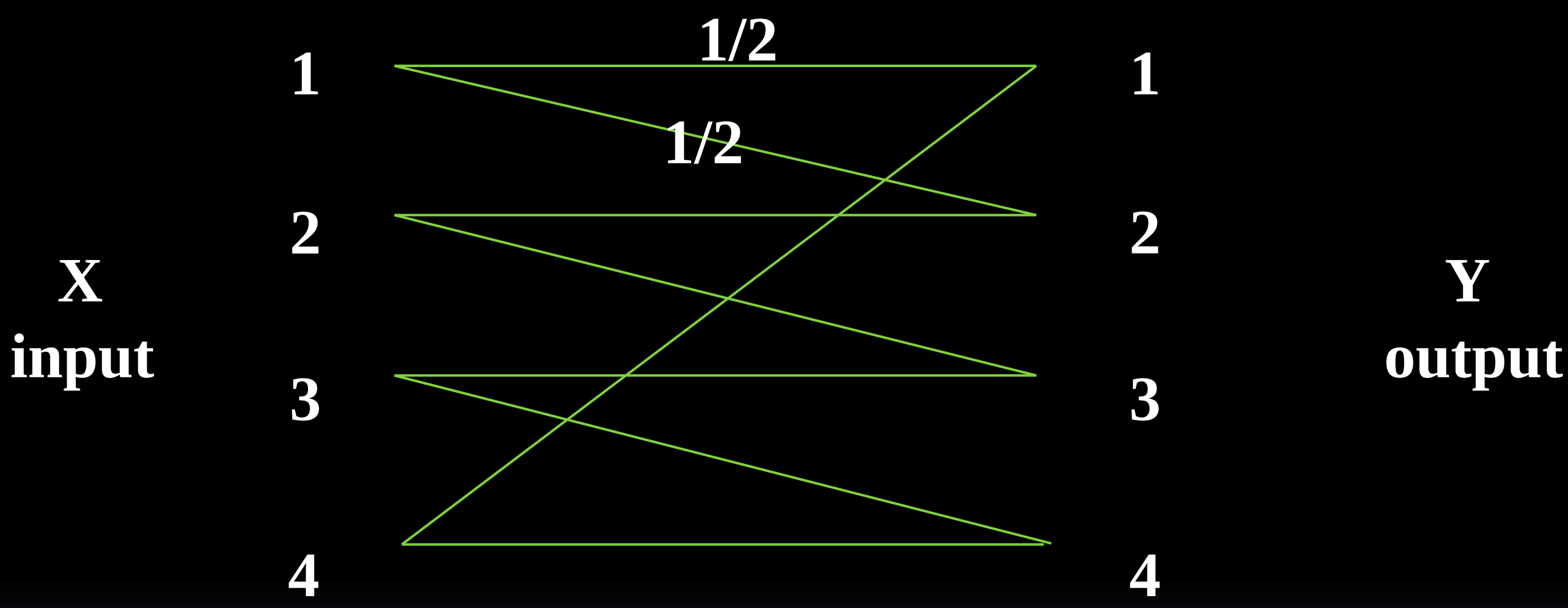
- DISTRIBUTED SOURCE CODING
  - BINNING VS QUANTIZATION
  - COSET CODES

- DISCUSSION

Information Theory
Some seminal papers by Shannon

- Channel Coding, 1948
- Source Coding, 1948, 1958
- Cryptography, 1949

Channel coding - example

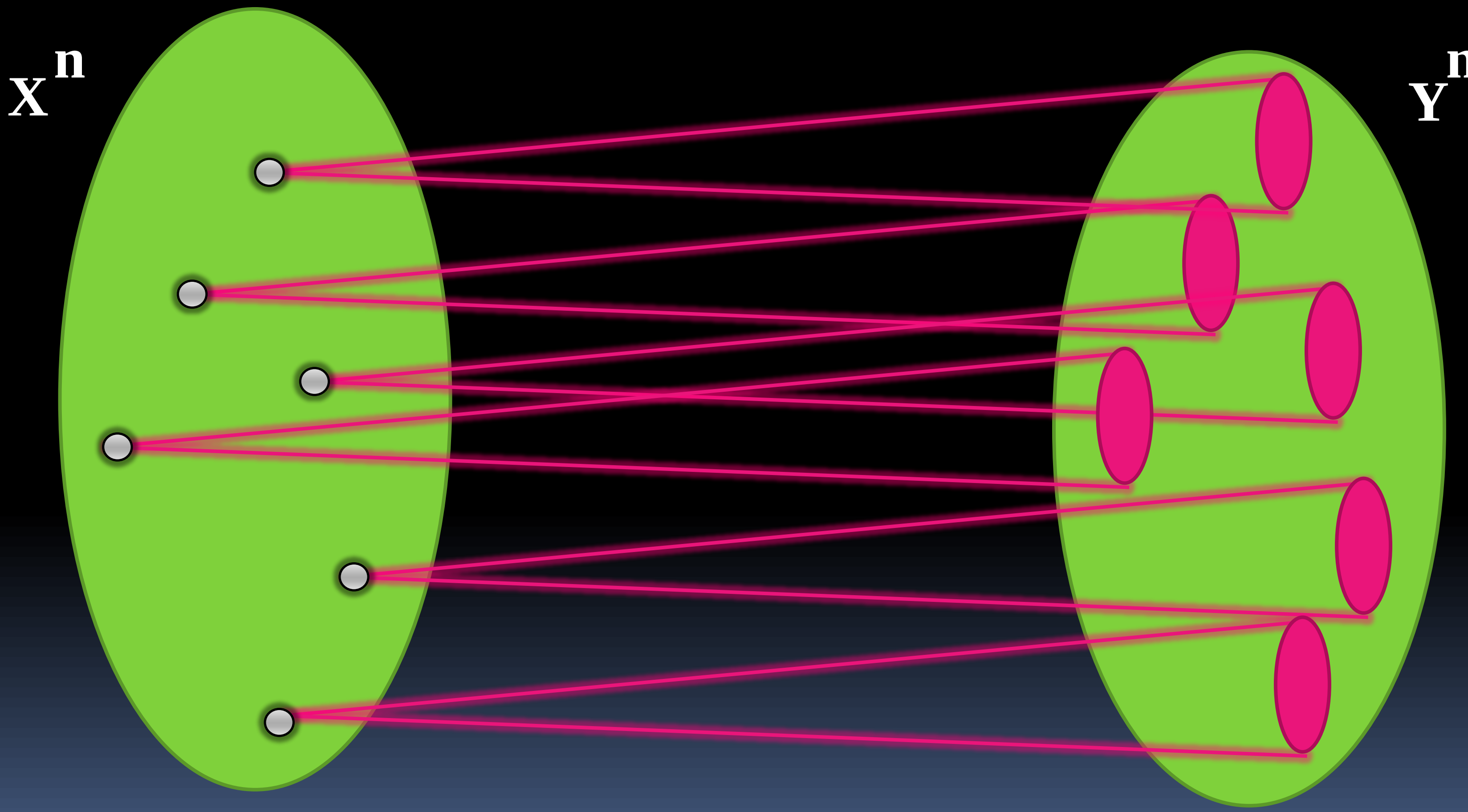


Capacity: 1 bit/transmission
Best code: Use only inputs {1 , 3}

Exercise moderation!!

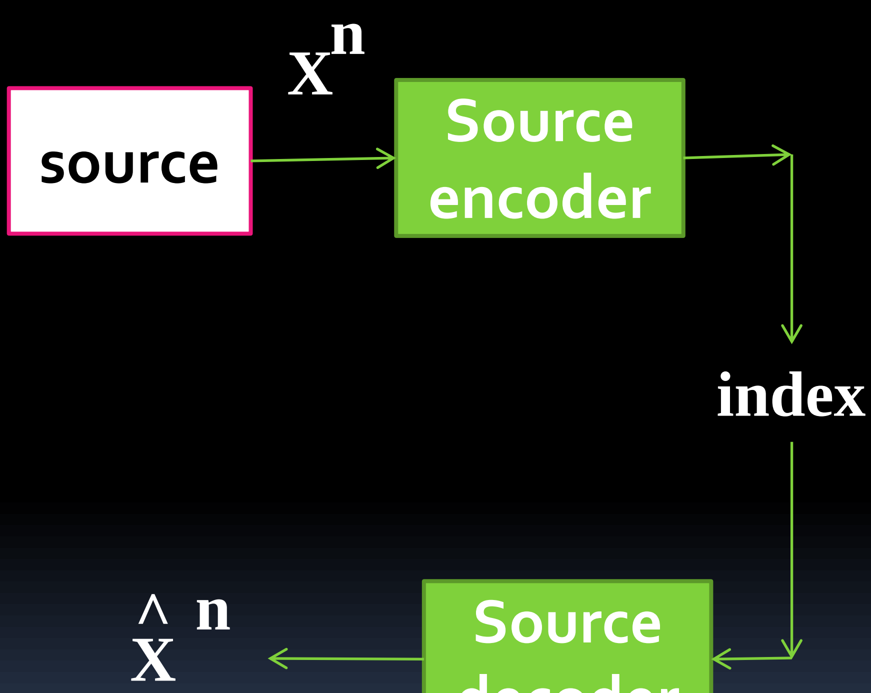
Channel coding

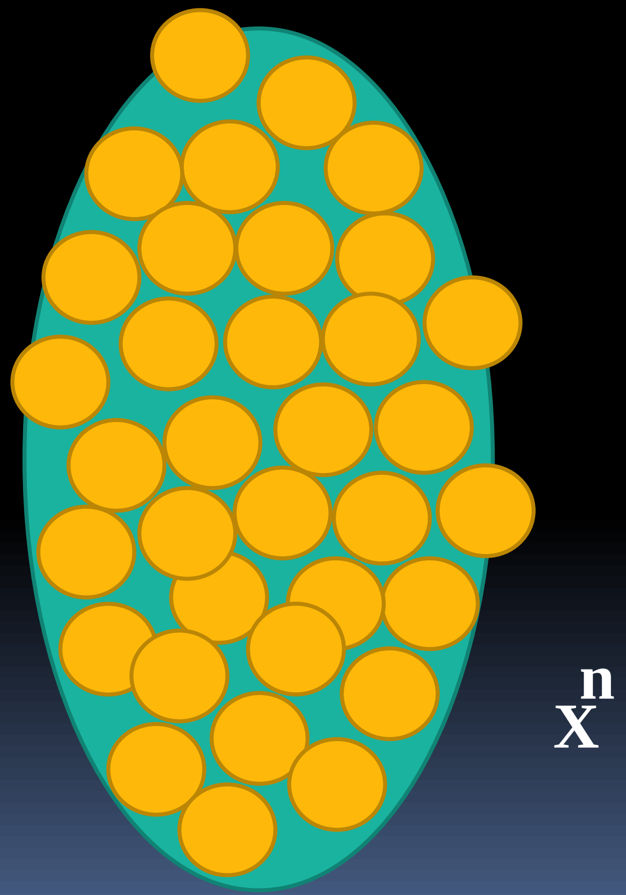
Typically need larger codes, $n >> 1$



Similar to sphere paching

Source coding:
Get good representation of source with few bits

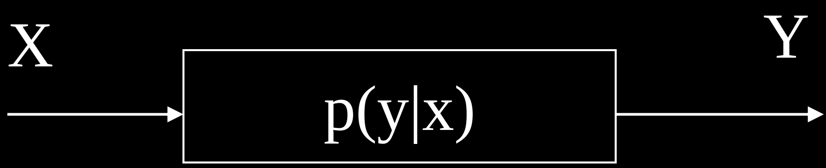




Similar to sphere covering

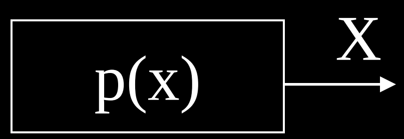
Introduction

CHANNEL CODING



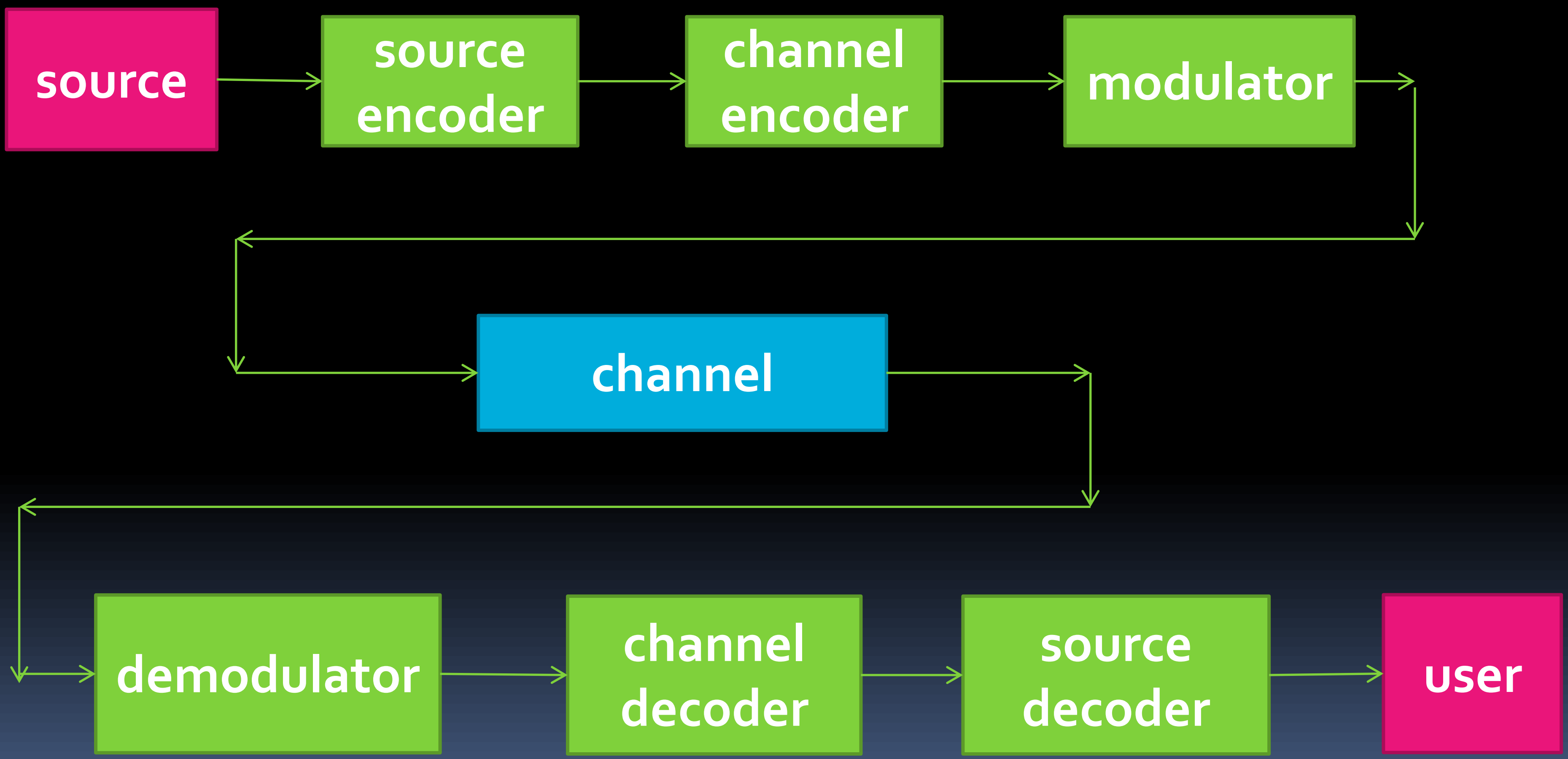
$$C = \max_{p(x)} I(X;Y)$$

SOURCE CODING



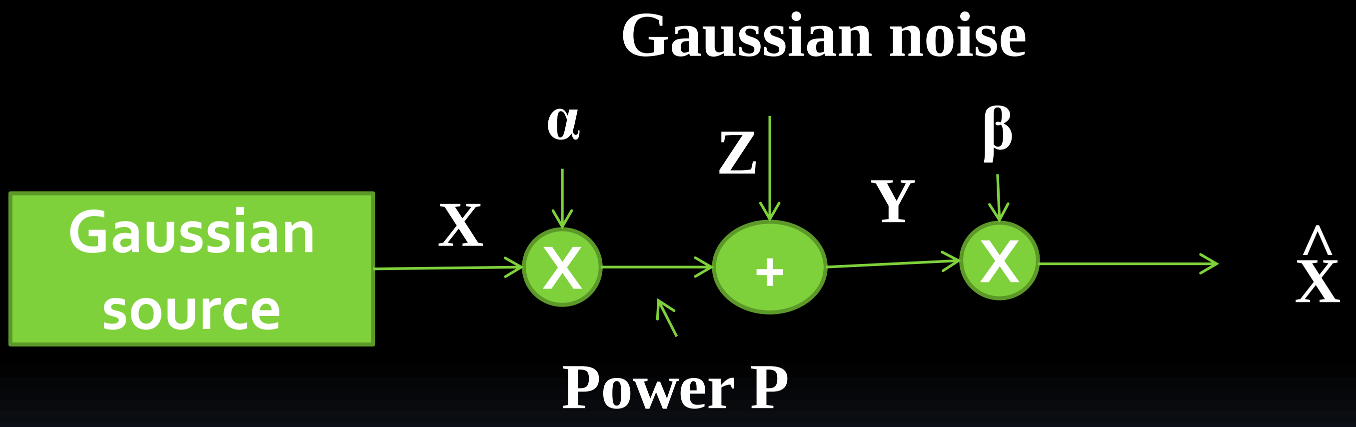
$$R(D) = \min_{p(\hat{x}|x)\,:\, E\, d(x,\hat{x})<D} I(X;\hat{X})$$

Source and channel coding in communication system

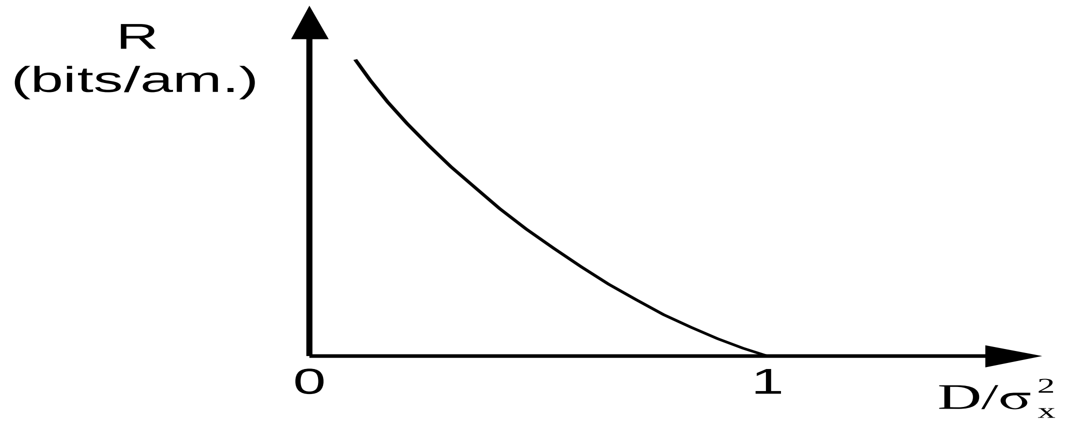


Joint source and channel coding

Can be simple if source and channel are matched



Rate distortion theory



Example: Gaussian source with memory

$$D(R) = 2^{-2R}\sigma_x^2$$

or

$$R(D) = \frac{1}{2}\log_2\left(\frac{\sigma_x^2}{D}\right)$$

$$\therefore \quad Max\ SNR(dB) = 10\log_{10}\left(\frac{\sigma_x^2}{D(R)}\right) = 20R\log_{10} 2 \cong 6R$$

RMS distortion

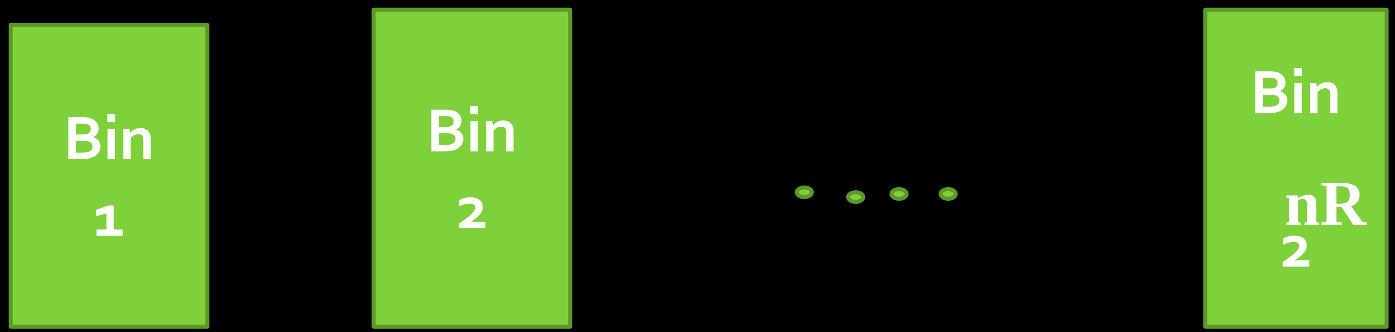
Dirty paper coding

CODING FOR MEMORIES WITH DEFECTS

1 0 0 1 1 0

“stuck-at” defects – probability α

<u>Binning:</u> Randomly distribute all 2^n sequences into 2^{nR} “bins”



$$\text{\# of sequences in each bin} = \frac{2^{n}}{2^{nR}} = 2^{n(1-R)}$$

$$E\,(\text{\# of matching sequences in a bin}) = 2^{n(1-R)} \cdot 2^{-n\alpha}$$

$$= 2^{n(1-\alpha-R)}$$

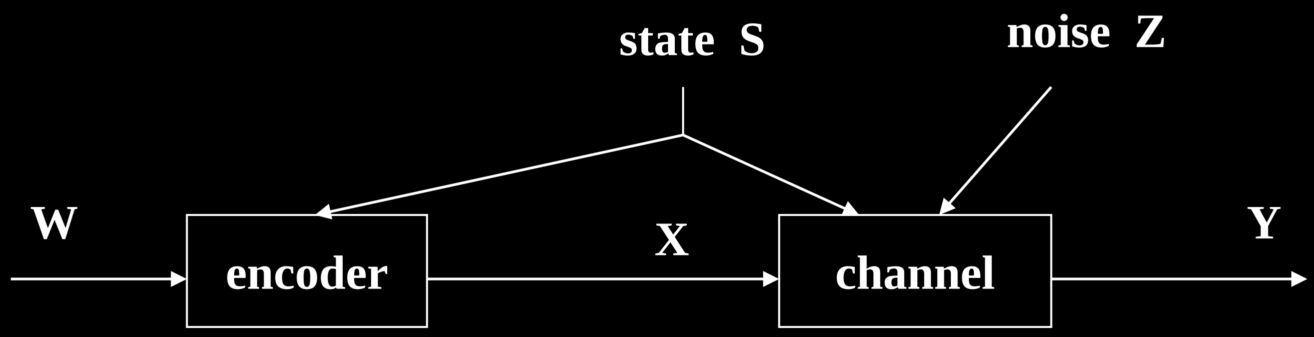
Note:

If $R < 1-\alpha \rightarrow$ guaranteed to have a match

Thus Capacity = $1-\alpha$ bits per memory cell

(same as if receiver knew defect positions)

Model

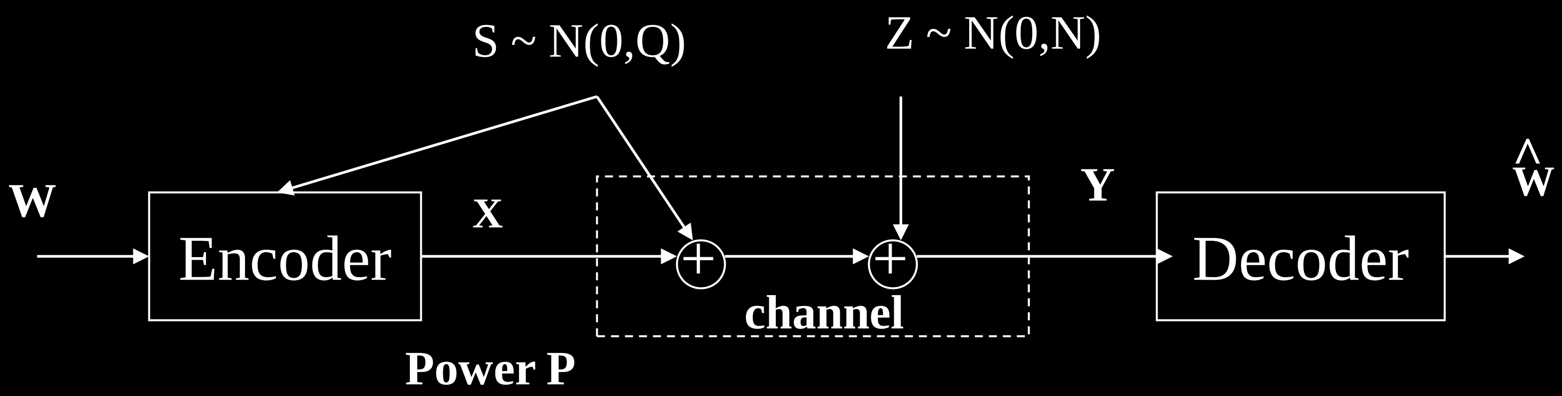


General solution (Gelfand and Pinsker):

$$C = \max_{p(u,x|s)} \left(I(U;Y) - I(U;S) \right)$$

In the example: U = Y

Analog version



$$C = \max_{p(u,x|s)} \left(I(U;Y) - I(U;S) \right)$$

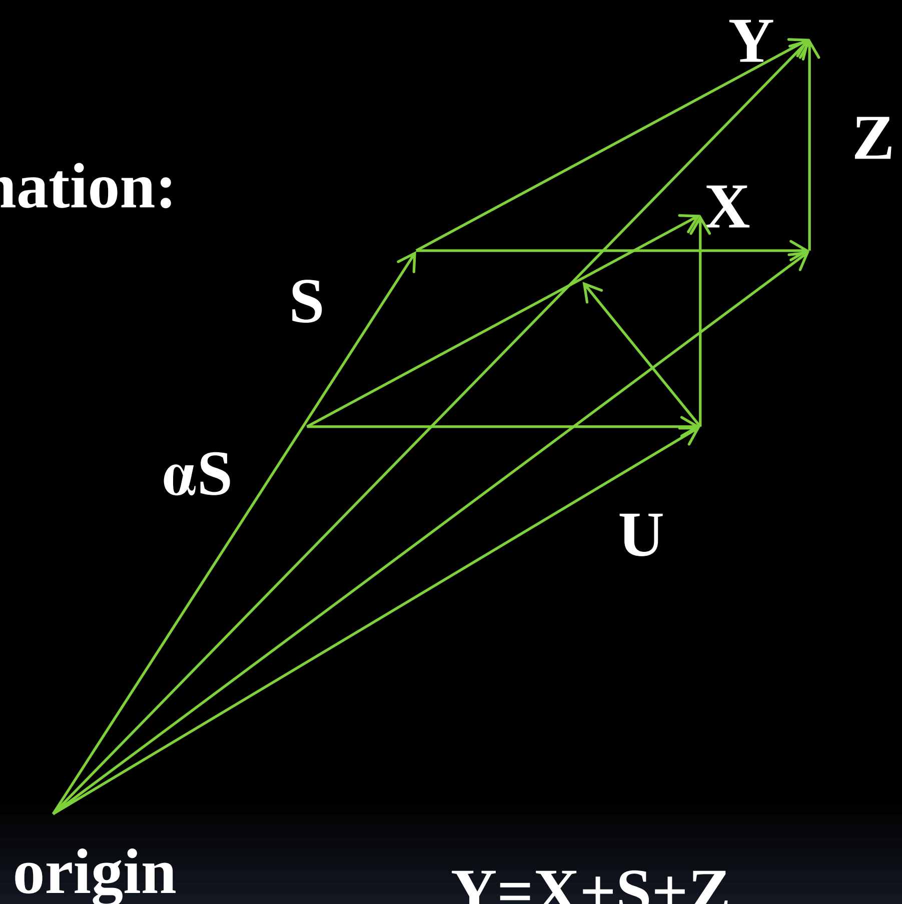
Adopt $U = X + \alpha S$, maximize over α

Result:

$$C = \tfrac{1}{2} \log \left(1 + P / N \right), \quad \text{independently of } Q$$

Obtained with $\alpha = P / (P+N)$

Geometrical explanation:

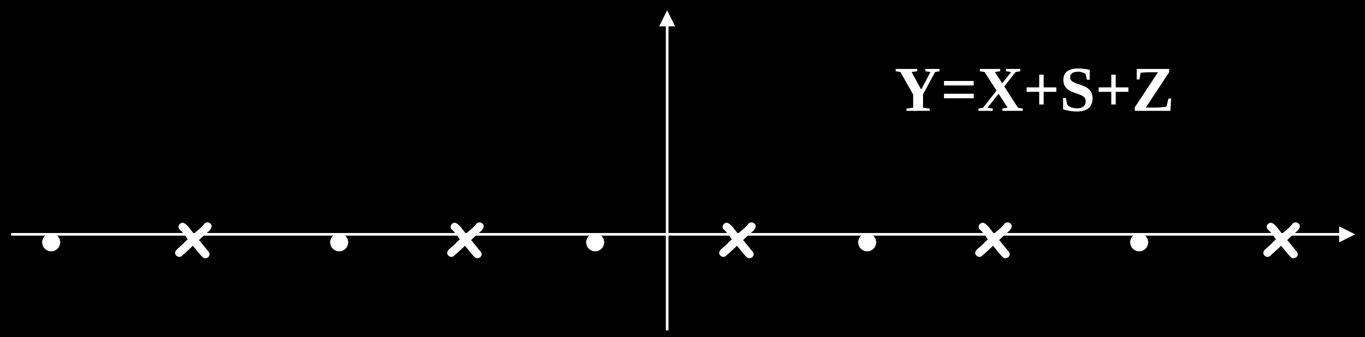


$Y = X + S + Z$

$U = X + \alpha S$

Approximate methods

QIM (QUANTIZATION INDEX MODULATION)

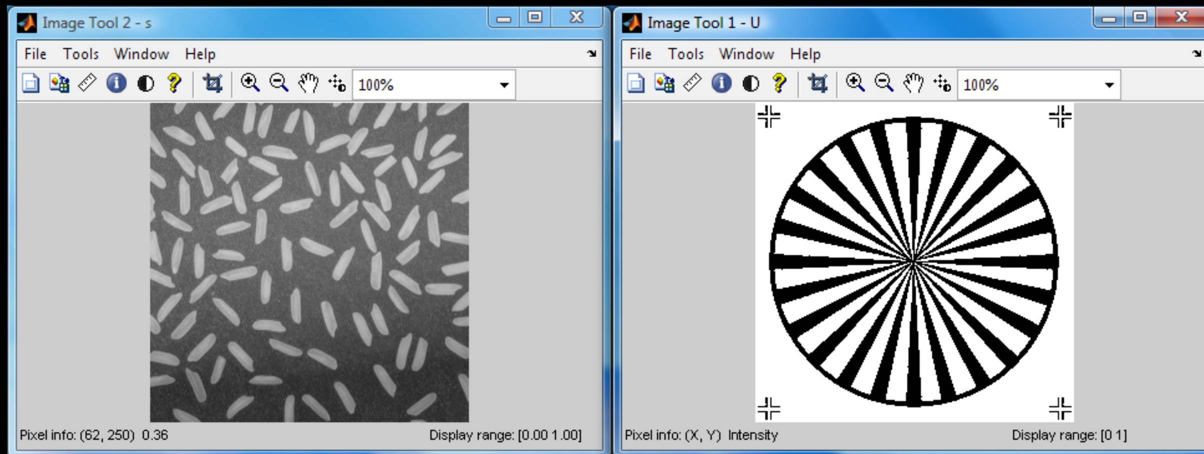


$f_{\Delta}(y) = \mathrm{mod}\,(y+\Delta/2\,,\,\Delta) - \Delta/2$

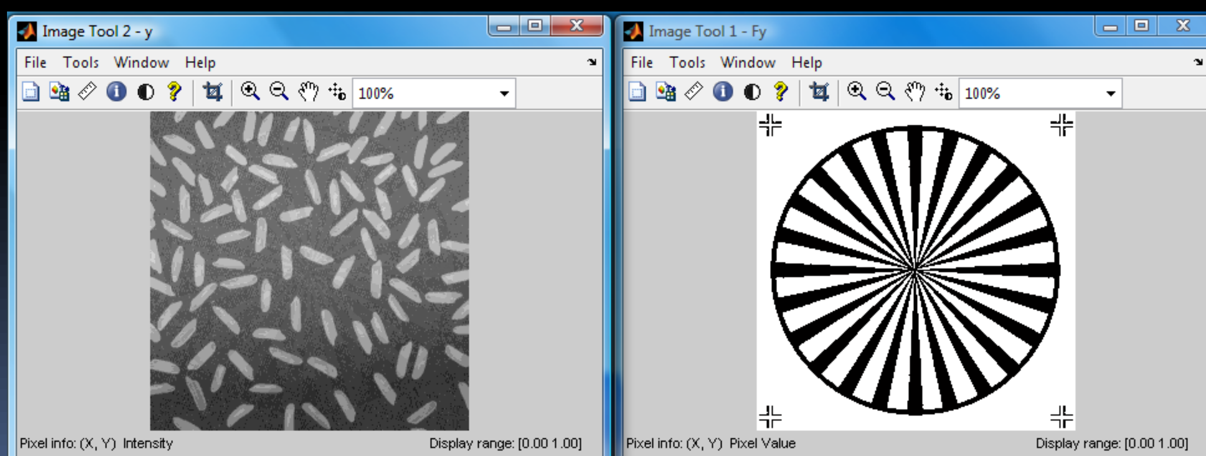
Encoding: $X = f_{\Delta}(U\text{-}S) = U - S - k\,\Delta,$ k integer

Decoding: $\hat{W} = f_{\Delta}(Y) = f_{\Delta}(U\text{-}S\text{-}k\,\Delta + S + Z) = f_{\Delta}(U + Z)$

Watermark example



Images for watermark and host signal



Images for received watermark and received signal

Variations

- PARTITIONED LINEAR BLOCK CODES (HEEGARD, 1983)
- COSET CODES (FORNEY, RAMCHANDRAN)
- APPLICATIONS WITH BCH CODES, REED-SOLOMON CODES
- APPLICATIONS WITH LATTICES
- APPLICATIONS WITH LDPC, LDGM

Distributed source coding

$$R(D) = \min_{p(w|x)\,:\,Ed(X,\hat{X})<D} \left(I(X;W) - I(Y;W) \right)$$

Simple example

- X and Y vectors of size 3
- Hamming distance ≤ 1

- **Case 1:** Y known by all (i.e., encoder and decoder)

  R = H(X|Y) = 2 bits (just send X+Y)

- **Case 2:** Y known only by decoder – use coset codes

  Here too R = 2 bits

  Send index of coset of X (use a repetition code)

  Decoding: Using coset of X and Y, recover X exactly

Repetition code – standard array

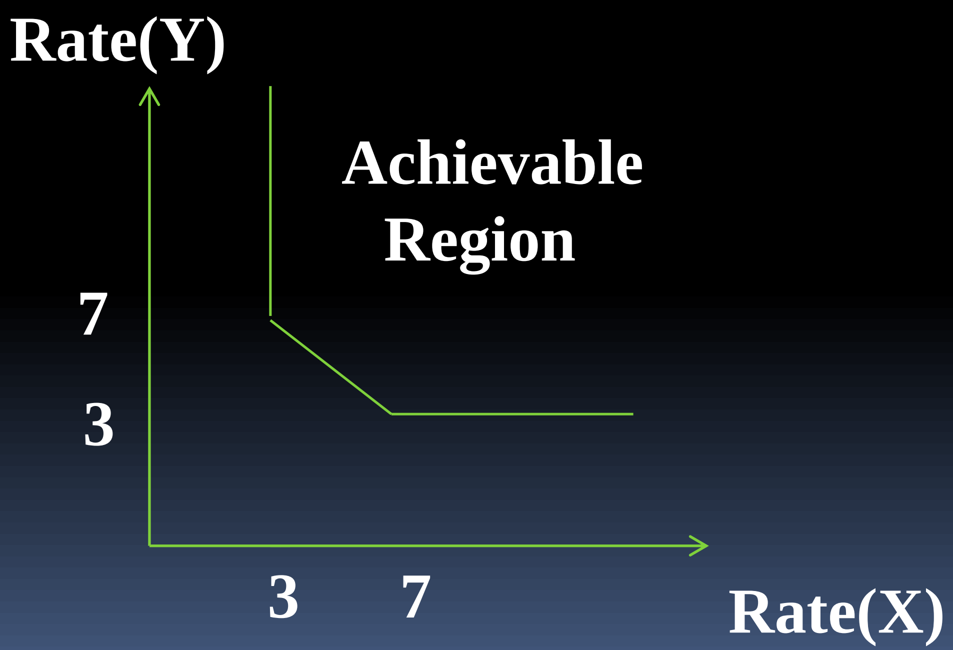
Code (coset 0)	000	111
Coset 1	001	110
Coset 2	010	101
Coset 3	100	011

Can get X from Y and coset number

Another simple example:

Let X and Y be unif. distributed length 7 binary sequences
Hamming distance (X,Y) ≤ 1

H(X) = H(Y) = 7 bits
H(X|Y) = H(Y|X) = 3 bits
H(X,Y) = 10 bits



Encoding: use 3 bits (8 possible cosets)

To encode X use coset of a Hamming (7,4) code

Decoding:

Based on coset number and on Y, find X

Binning operation

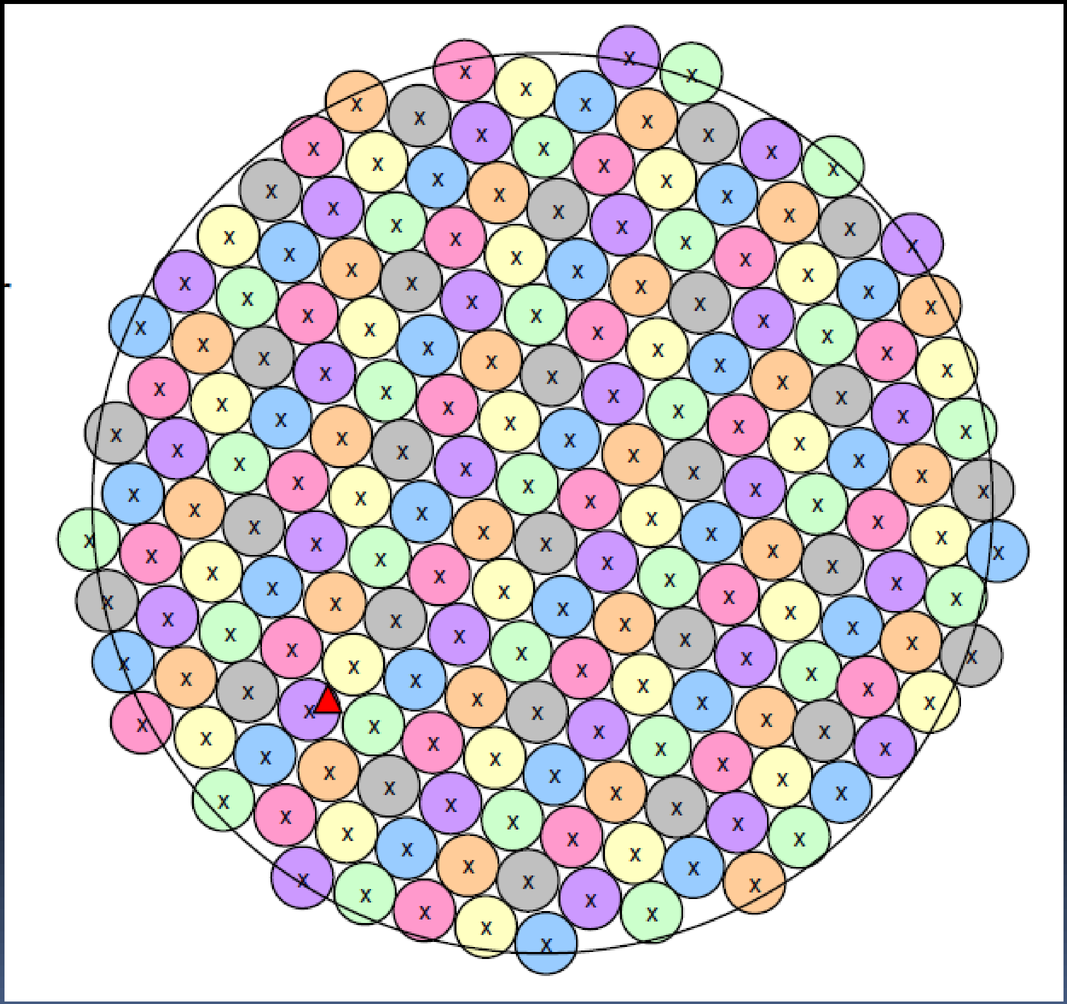


Figure credit: K. Ramchandran

Dual operations

- **QUANTIZATION**

  integer division resulting in quotient

- **BINNING**

  integer division resulting in remainder

Applications

- DIGITAL WATERMARKING
- STEGANOGRAPHY
- CELLULAR TELEPHONY (DOWNLINK)
- COGNITIVE RADIO
- RADIO BROADCASTING
  - DIGITAL-TV OVER ANALOG-TV (CHINOOK COMM., BOSTON AREA)
  - DIGITAL RADIO OVER FM RADIO (ALTERNATIVE TO IBOC AND DRM)
- VIDEO COMPRESSION (DISTRIBUTED SOURCE CODING)
- VIDEO SYNCHRONIZATION

Information theory is alive and well !